DOCTORAL (PHD) STUDIES
COURSE DESCRIPTION

Course title	Field of science	Faculty	Institute
Theory of Navier-Stokes Equations	Mathematics (N 001)	Faculty of Mathematics and Informatics	Institute of Applied Mathematics
Study method	Number of credits	Study method	Number of credits
Lectures	0	Consultations	1
Individual work	4	Seminars	0

Course summary
Auxiliary results. The spaces of divergence free vector fields. Divergence problem. Interpolation and multiplicative inequalities. Stokes system. Definitions of week solutions. Existence and uniqueness of a week solution in a bounded domain. Regularity of a week solution. The spectrum of the Stokes operator. Stokes system with non-homogeneous boundary condition. Stationary Navier-Stokes system with homogeneous boundary condition. Stationary Navier-Stokes system with non-homogeneous boundary condition. Regularity of week solutions to the Navier-Stokes system. Nonstationary Stokes system. Definitions of week solutions. Existence and uniqueness of a week solution. Regularity of week solutions. Nonstationary Navier-Stokes system. Existence and uniqueness of regular solutions in the two-dimensional case. Existence and uniqueness of regular solutions in the three-dimensional case. Week solutions of the nonstationary Navier-Stokes problem. The flow of viscous incompressible fluid in infinite cylinder. Stationary Poiseuille solution. Definition of the nonstationary Poiseuille type solutions. Construction of an approximate solution to the inverse problem. Existence and uniqueness of the solution to the inverse problem in Holder and Sobolev spaces. The behavior of the nonstationary Poiseuille solution as $t \to \infty$. Time-periodic Poiseuille solution.
Main literature
1. K. Pileckas. *Navjė-Stokso lygčių matemaitinė teorija*. MII, Vilnius, 2007.
2. O. A. Ладыженская. *Математические вопросы динамики вязкой жидкости*. Наука, Москва, 1970.
3. P. Temam. *Navier-Stokes Equations, Theory and Numerical Analysis.* North-Holland, Amsterdam, 1979.
4. G. P. Galdi. *An Introduction to the Mathematical Theory of Navier-Stokes Equations*. v. I and II. Springer Tracts in Nat. Ph., 38, 39. Springer-Verlag, 1994.
5. H. Sohr. *The Navier-Stokes Equations: An Elementary Functional Analytic Approach*. Birkhauser, Basel, 2001.
6. G. K. Batchelor. *An Introduction to Fluid Dynamics*. Cambridge University Press, Cambridge, 2002.
7. Л. Д. Ландау, Е. М. Лифшиц. *Гидродинамика*. Наука, Москва, 1988.

Consulting teacher	Scientific degree	Pedagogical name	Main publications in the field of science of the last 5 year period
Konstantinas Pileckas	Habil. dr.	Prof.	1. Korobkov, Mikhail; **Pileckas, Konstantinas**; Russo, Remigio. Leray's plane steady state solutions are nontrivial, *Advances in Mathematics, Volume 376,* 2021. DOI:10.1016/j.aim.2020.107451 2. **Pileckas, Konstantinas,** Raciene, Alicija.Non-stationary Navier-Stokes equations in 2D power cusp domain. I. Construction of the formal asymptotic decomposition, *Advances in Nonlinear Analysis, vol. 10, no. 1, 2021, pp. 982-1010,* 2021. DOI: 10.1515/anona-2020-0164. 3. **Pileckas, Konstantinas,** Raciene, Alicija. Non-stationary Navier-Stokes equations in 2D power cusp domain. II. Existence of the solution, *Advances in Nonlinear Analysis, vol. 10, no. 1, 2021, pp. 1011-1038*, 2021. DOI: 10.1515/anona-2020-0165. 4. Korobkov, Mikhail; **Pileckas, Konstantinas**; Russo, Remigio. On the steady Navier-Stokes equations in 2D exterior domains, *Journal of Differential Equations,* **269**, iss. 3, (2020) 1796-1828. DOI: 10.1016/j.jde.2020.01.012

			5. Korobkov, Mikhail; **Pileckas, Konstantinas**; Russo, Remigio. Solvability in a finite pipe of steady-state Navier-Stokes equations with boundary conditions involving Bernoulli pressure, *Calculus of Variations and Partial Differential Equations*, **59**, iss. 1, (2020) , art. no. 32, 1-22. DOI: 10.1007/s00526-019-1688-8. 6. Korobkov, Mikhail, V; **Pileckas, Konstantinas**; Russo, Remigio. On convergence of arbitrary D-solution of steady Navier-Stokes system in 2D exterior domains, *Archive for Rational Mechanics and Analysis*, **233,** iss. 1, (2019), 385-407. DOI: 10.1007/s00205-019-01359-8.

Approved by the Board of Faculty of Mathematics and Informatics 10/12/2021. Resolution No. (1.5 E) 110000-TPN-42
Board Chairman – assoc. prof. dr. Kristina Lapin