DOCTORAL STUDIES COURSE UNIT DESCRIPTION

Name of subject	Scientific Field	Center	Department
Nonlinear Dynamics, Bifurcation Theory and Chaos (8 ECTS credits)	Physics N 002	Center for Physical Sciences and Technology	Fundamental Research
Student's workload	Hours	Student's workload	Hours
Lectures		Consultations	20
Individual study	180	Seminars	

Course annotation
Qualitative and quantitative research methods of nonlinear dynamical systems are considered. The main concepts of nonlinear dynamical systems, such as a phase space, a fixed point, a limit cycle, stability and others are introduced. Asymptotic methods to solve nonlinear differential equations are studied: multiple-scale expansion method, averaging method and harmonic balance method. Various third-order chaotic dynamical systems and their physical models are discussed. A large part of the course is devoted the bifurcation theory. This theory examines the qualitative changes in a nonlinear dynamic system when the system control parameter is varied. When a nonlinear system is close to the bifurcation point, it can be described by a universal analytical model. Students should master the applications of nonlinear dynamic methods in solving specific problems of nonlinear dynamics. The aim of the course is to acquaint with modern research methods of nonlinear dynamical systems and to form practical skills of their application. The course is based on the following topics: qualitative and quantitative research methods of nonlinear dynamical systems; studies of phase portraits of various physical systems; fixed points and their classification; limit cycles, Liapunov functions, Puankare and Bendixon theorem, theory of self-oscillations: quasi-harmonic and relaxation oscillations, asymptotic methods (multiple scale method, averaging and harmonic balance methods), synchronization of nonlinear oscillators; non-autonomous systems: forced oscillations of a nonlinear oscillator, parametric resonance; bifurcation theory: saddle-node bifurcation, transcritical and pitchfork bifurcation, supercritical, subcritical and degenerate Hopf bifurcation, global limit cycle bifurcations: saddle-node bifurcation of limit cycles, infinite period bifurcation, homoclinic bifurcation; chaos theory: Lorentz system and other third-order chaotic systems, first- and second-order Poicare maps, topological characteristics of strange attractors, chaos occurrence scenarios, computer algorithms for determining strange attractor characteristics, synchronization of chaotic systems, control of chaos.
List of literature
1. A. Medio and M. Lines, Nonlinear Dynamics: A Primer (Cambridge University Press, 2001) 2. D. Kaplan and L. Glass, Understanding Nonlinear Dynamics (Springer-Verlag,1995) 3. S. Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos (Springer-Verlag,1990) 4. P. Glendinning, Stability, Instability and Chaos: an introduction to the theory of nonlinear differential equations (Cambridge University Press, 1994) 5. S.H. Strogatz. Nonlinear Dynamics and Chaos (Addison-Wesley, 1994). 6. J. Gukenheimer and P. Holmes. Nonlinear oscillations, dynamical systems and bifurcations of vector fields (Springer, 1983)

7. J. Kevorkian and J. D. Cole, Multiple scale and singular perturbation methods (Applied Mathematical Sciences) (Springer-Verlag 1996)
8. J. A Sanders, F. Verhulst and J. Murdock, Averaging Methods in Nonlinear Dynamical Systems (Springer, 2007)
9. Y. A. Kuznetsov, Elements of Applied Bifurcation Theory (Springer-Verlag,1998)
10. H. G. Schuster and W. Just, Deterministic Chaos, An Intruduction (Willey-VCH, 2005)
11. E. Ott, Chaos in Dynamical Systems (Cambridge University Press, 1993)
12. A. B. Cambel, Applied Chaos Theory: A Paradigm for Complexity (Academic Press, 1993)
13. A. Scott (ed.) Encyclopedia of Nonlinear Science (Routlinge New York and London, 2005)
14. K. T.Alligood, T. Sauer and J.A.Yorke, Chaos: an introduction to dynamical systems (Springer-Verlag, 1997)
15. R. L. Devaney, An Introduction to Chaotic Dynamical Systems (Westview Press, 2003)
16. R. Hilborn, Chaos and Nonlinear Dynamics: An Introduction for Scientists and Engineers (Oxford University Press, 2000)

Consulting teachers	Scientific degree	Pedagogical name	Main scientific works published in a scientific field in last 5 year period
Kęstutis Pyragas	habil. Dr.	Prof.	1. I. Ratas and K. Pyragas, *Macroscopic self-oscillations and aging transition in a network of synaptically coupled quadratic integrate-and-fire neurons*, Phys. Rev. E 94, 032215 (2016). 2. V. Pyragas and K. Pyragas, *Act-and-wait time-delayed feedback control of nonautonomous systems*, Phys. Rev. E 94, 012201 (2016). 3. I. Ratas and K. Pyragas, *Eliminating synchronization in bistable networks*, Nonlinear dynamics 83, 1137-1151 (2016). 4. K. Pyragas and P.A. Tass, *Suppression of spontaneous oscillations in high-frequency stimulated neuron models*, Lith. J. Phys. 56, 223–238 (2016). 5. T. Pyragienė and K. Pyragas, *Anticipatory synchronization via low-dimensional filters*, Phys. Lett. A 381, 1893-1898 (2017). 6. I. Ratas and K. Pyragas, *Symmetry breaking in two interacting populations of quadratic integrate-and-fire neurons*, Phys. Rev. E 96, 042212, (2017). 7. K. Pyragas, A.P. Fedaravičius, T. Pyragienė and P.A. Tass, *Optimal waveform for entrainment of a spiking neuron with minimum stimulating charge*, Phys. Rev. E 98, 042216 (2018). 8. V. Pyragas and K. Pyragas, *Act-and-wait time-delayed feedback control of*

			autonomous systems, Phys. Lett. A 382, 574-580 (2018). 9. I. Ratas and K. Pyragas, *Macroscopic oscillations of a quadratic integrate-and-fire neuron network with global distributed-delay coupling*, Phys. Rev. E 98, 052224 (2018). 10. T. Pyragienė and K. Pyragas, *Design of a negative group delay filter via reservoir computing approach: Real-time prediction of chaotic signals*, Phys. Lett. A 383, 3088-3094 (2019). 11. V. Pyragas and K. Pyragas, *State-dependent act-and-wait time-delayed feedback control algorithm*, Communications in nonlinear science and numerical simulation 73, 338-350 (2019). 12. I. Ratas and K. Pyragas, *Noise-induced macroscopic oscillations in a network of synaptically coupled quadratic integrate-and-fire neurons*, Phys. Rev. E 100, 052211 (2019). 13. K. Pyragas, A.P. Fedaravičius, T. Pyragienė and P.A. Tass, *Entrainment of a network of interacting neurons with minimum stimulating charge,* Phys. Rev. E 102, 012221 (2020). 14. V. Pyragas and K. Pyragas, *Using reservoir computer to predict and prevent extreme events*, Phys. Lett. A 384, 126591 (2020).
Certified during Doctoral Committee session 02/02/2022, protocol No. (7.17 E) 15600-KT-32			
Committee Chairman prof. S. Juršėnas			