**DOCTORAL (PHD) STUDIES**
**COURSE DESCRIPTION**

| Course title | Field of science | Faculty | Institute |
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| **Equations of Mathematical Physics** | Mathematics (N 001) | Faculty of Mathematics and Informatics | Institute of Applied Mathematics |
| Study method | Number of credits | Study method | Number of credits |
| Lectures | 0 | Consultations | 1 |
| Individual work | 4 | Seminars | 0 |

| Course summary |
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| 1. The theory of Sobolev spaces. Generalized derivatives and their properties. Definitions of Sobolev spaces. Extension of functions from Sobolev spaces. Integral operators with weekly singular kernel. Embedding theorems. Equivalent norms in Sobolev spaces. Interpolation and multiplicative inequalities. Traces of functions from Sobolev spaces.<br>2. Elliptic equations. Dirichlet boundary value problem. Definition of week solution. Existence and uniqueness of week solution. Regularity of week solution. The spectrum of formally self-adjoint operator. Different boundary value problems. Strongly elliptic systems.<br>3. Parabolic equations. Formulation of the initial boundary value problem. Dirichlet problem. Definitions of week solutions. Existence and uniqueness of week solutions. Poof of the existence by Fourier method. Poof of the existence by Galerkin method. Regularity of week solutions. The case of different boundary conditions. Cauchy problem.<br>4. Hyperbolic equations. Formulation of the initial boundary value problem. Dirichlet problem. Definitions of week solutions. Existence and uniqueness of week solutions. Fourier and Galerkin methods. The case of different boundary conditions. Cauchy problem.<br>5. Variational methods. Semi-continuous from below functional. Minimum of non-differentiable functionals. Differentiation of nonlinear functionals. Minimum of differentiable functionals. Subgradient and subdifferential. Minimizing sequences. Dirichlet principle.<br>Applications: Neumann problem, minimum quadratic functional. Variational inequalities. Variational inequalities in Hilbert spaces. Non-coercive variational inequalities. Monotone operators. Variational inequalities with nonlinear monotone operators.<br>Topological methods. Banach fixed point theorem. Browder fixed point theorem. Schauder fixed point theorem. Leray-Schauder fixed point theorem. Extension by parameter method.<br>Generalized functions. Sequences of Delta-type functions. Properties of generalized functions. Series of generalized functions. Fundamental solutions of differential operators. Fourier transforms of generalized functions. |
| Main literature |
| 1. A. Ambrazevičius, A. Domarkas. *Matematinės fizikos lygtys*, 2 dalys Aldorija, Vilnius, 1999. |
| 2. О. А. Ладыженская. *Краевые задачи математической физики*. Наука, Москва, 1973 |
| 3. О. А. Ладыженская, Н. Н. Уральцева. *Линейные и квазилинейные уравнения эллиптического типа*. Наука, Москва, 1973 |
| 4 О. В. Бесов, В. П. Ильин, С. М. Никольский. *Интегральные представления функций и теоремы вложения.* Наука, Москва, 1975 |
| 5. R. A. Adams. *Sobolev Spaces*. Academic Press, New York, San Francisco, London, 1975 |
| 6. L. C. Evans. *Partial Differential Equations*. Graduate Studies in Mathematics, 19, Providence, 1991 |
| 7. В. С. Владимиров. *Обобщенные функции в математической физике*. Наука, Москва, 1979 |

| Consulting teacher | Scientific degree | Pedagogical name | Main publications in the field of science of the last 5 year period |
|---|---|---|---|
| Konstantinas Pileckas | Habil. dr. | Prof. | 1. Čiegis, Raimondas; Panasenko, Grigory**; Pileckas, Konstantinas**; Šumskas, Vytenis. ADI scheme for partially dimension reduced heat conduction models, *Computers & Mathematics with Applications*, **80**, iss.5, (2020) 1275-1286. DOI: 10.1016/j.camwa.2020.06.012<br>2. **Pileckas, Konstantinas;** Čiegis, Raimondas. Existence of nonstationary Poiseuille type solutions under minimal regularity assumptions*, Zeitschrift für angewandte Mathematik und Physik (ZAMP),* **71**, iss. 6 (2020), art. no. 192, p. 1-17. DOI: 10.1007/s00033-020-01422-5. |

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| | | | 3. Panasenko, Grigory; **Pileckas, Konstantinas.** Periodic in time flow in thin structure: Equation the graph, *Journal of Mathematical Analysis and Applications,* **490**, iss. 2, (2020), art. no. 124335, p. 1-8.<br>DOI: 10.1016/j.jmaa.2020.124335.<br>4. Bertoglio, Cristobal; Conca, Carlos; Nolte, David; Panasenko, Grigory; **Pileckas, Konstantinas**. Junction of models of different dimension for flows in tube structures by Womersley-type interface conditions, *SIAM Journal on Applied Mathematics,* **9**, iss 3, (2019) 959-985.<br>DOI: 10.1137/18M1229572.<br>5. Korobkov, Mikhail; **Pileckas, Konstantinas**; Russo, Remigio. The existence theorem for the steady Navier-Stokes problem in exterior axially symmetric 3D domains, *Mathematische Annalen*, **370,** iss. 1-2, (32018), 727-784.<br>DOI: 10.1007/s00208-017-1555-x. |

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| Approved by the Board of Faculty of Mathematics and Informatics 10/12/2021. Resolution No. (1.5 E) 110000-TPN-42 |
| Board Chairman – assoc. prof. dr. Kristina Lapin |