DOCTORAL (PHD) STUDIES
COURSE DESCRIPTION

Course title	Field of science	Faculty	Institute
Insurance Mathematics	Mathematics (N 001)	Faculty of Mathematics and Informatics	Institute of Mathematics, Institute of Applied Mathematics
Study method	Number of credits	Study method	Number of credits
Lectures	0	Consultations	1
Individual work	4	Seminars	0

Course summary

Non-Life insurance

Static aspects of insurance theory. Collective risk system. Risk portfolio. Static insurance model. Insurance in terms of utility theory. Principles of premium calculation. Panjer‘s formulas to get distribution of sum of discrete distributions. Distribution classes (a, b, 0) and (a, b, 1). Schroter‘s distribution class. Approximate formulas for probabilities of convolutions.

Dynamic models of insurer stock development. Discrete time risk process. Classical risk process. Risk renewal process. Investment risk processes.

Poisson process. Homogeneous and inhomogeneous Poisson processes. The main properties of these processes.

Renewal process. Generating of the renewal process. The law of large numbers. Numerical characteristics of the renewal process. Asymptotic behavior of these characteristics. Renewal equation and its solution. W. Smith‘s and Blackwell‘s theorems. Central limit theorem for the renewal process.

Other claims number processes. Cox, mixed Poisson and Polya processes. General requirements for claims number process.

Classification of claims. Small and large claims. Heavy-tailed distribution classes **R, L, D, C, S**. Relationships between these classes.

Sums of claims. The process of the claim sums and its properties. Asymptotic behavior of the numerical characteristics of the claim sums process. Central limit theorem for the claim sums process.

Ruin probability in the discrete time risk model. Recursive formulas for calculating ruin probabilities and finite-time ruin probabilities. A method of generating functions to find the exact expressions of bruin probabilities. Lundberg inequality. Martingale methods of assessing the ruin probability. Asymptotic formulas for ruin probability and finite-time ruin probability.

Ruin probability in the classical risk model. Defective renewal equation for for ruin probability. Expression of the ruin probability by compound geometric distribution. Applications of the Laplace-Stieltjes transforms to the solution of the renewal equation. Lundberg inequality. Martinghal methods of assessing the ruin probability. Asymptotic formulas of ruin and finite-time ruin probabilities in the classical risk model.

Ruin probability in the E.S. Andersen‘s risk renewal model. Net profit condition. Defective renewal equation for ruin probability. Expression of the ruin probability by the subordinated convolutions sum of the leader heights distributions. Equilibrium coefficient of the model. The Lundberg inequality for the ruin probability. Martingale methods to the ruin probability assessment. Asymptotics of ruin probability for small claims. Asymptotic formulas for ruin and finite-time ruin probabilities in the case of large claims.

Reinsurance. Reinsurance from the point of view of utility theory. Influence of reinsurance on the ruin probability. Insurers' unions and risk sharing. Reinsurance of the risk portfolio.

Gerber-Shiu discounted penalty function. Algorithms for calculating values of the penalty function in a discrete time risk model. Expressions of penalty functions in the classical risk model. The asymptotic properties of the penalty function for small and large claims.

Life insurance

Survival analysis. The main concepts in survival analysis. Analytical survival functions. Life tables. Classical methods for the survival function approximation. Population density function. Methods to describe this function. Lee-Carter‘s method.

Elements of a static life insurance model. Valuation of insurance benefits for different insurance contracts. Relations between values of continuous and discrete benefits. Recursion formulas for values of discrete benefits. Continuous and discrete annuities. Expected present values of continuous and discrete annuities for different insurance contracts.

Premium calculation methods for life insurance contracts.

Policy values with continuous and discrete cash flows.

Pension plans. Multi-factor model. Annual payment rate. Various expressions of this rate. Actuarial value of future retirement benefits in a multi-factor model.

Pension funds. Sorting populations by population density function. Pension fund characteristics for active members. Fund characteristics for passive fund members. Generalized characteristics of the pension fund. Calculation of these characteristics.

Options in life insurance.

Main literature
1. P. Embrechts, C. Klüppelberg, and T. Mikosch. *Modelling Extremal Events for Insurance and Finance*. Springer, Berlin, 1997.
2. J. Grandel. *Aspects of Risk Theory*. Springer, New York, 1991.
3. E. Straub. *Non-life Insurance Mathematics*. Springer, Berlin, 1988.
4. T. Mikosch. *Non-Life Insurance Mathematics*. Springer. 2009.
5. B. Benjamin, J. H. Pollard. *The Analysis of Mortality and Other Actuarial Statistics*. Butterworth-Heinemann. 1980.
6. N. L. Bowers et al. *Actuarial Mathematics*. Itasca. 1980.
7. H. U. Gerber. *Life Insurance Mathematics*. Springer. 1995.
8. D. C. M. Dickson, M. R. Hardy, H. R. Waters. *Actuarial Mathematics for Life Contingent Risks.* Cambrige University Press. 2009.
9. D. C. M. Dickson. *Insurance Risk and Ruin.* Cambridge University Press. 2005.
10. G. E. Willmott, X. S. Lin. *Lundberg Approximations for the Compound Distributions with Insurance Applications*. Springer. 2001.

Consulting teacher	Scientific degree	Pedagogical name	Main publications in the field of science of the last 5 year period
Remigijus Leipus	Habil. dr.	Prof.	1. **R. Leipus,** J. Šiaulys, On the random max-closure for heavy-tailed random variables. *Lithuanian Mathematical Journal*, 2017,57,208-221. 2. A. Buteikis, **R. Leipus**, An integer-valued autoregressive process for seasonality. *Journal of Statistical Computation and Simulation,* 2020, 90, 391-411. 3. **R. Leipus**, A. Philipppe, V. Pilipauskaitė, D. Surgailis, Estimating long memory in panel random-coefficient AR(1) data. *Journal of Time Series Analysis,* 2020, 41,520-535. 4. **R. Leipus**, J. Šiaulys, On a closure property of convolution equivalent class of distributions. *Journal of Mathematical Analysis and Applications,* 2020, 490, 124226. 5. S. Jokubaitis, D. Celov, **R. Leipus**, Sparse structures with LASSO through principal components: Forecasting GDP components in the short-run. *International Journal of Forecasting*, 2021, 37, 759-776.

Jonas Šiaulys	Dr. (HP)	Prof.	1. E. Bernackaitė, **J. Šiaulys**, The finite-time ruin probability for an inhomogeneous renewal risk model. *Journal of Industrial and Management Optimization,* 2017, 13, 207-222. 2. S. Danilenko**, J. Šiaulys**, G. Stepanauskas, Closure properties of O-exponential distributions. *Statistics and Probability Letters,* 2018, 140, 63-70. 3. O. Ragulina, **J. Šiaulys**, Randomly stoped minima and maxima with exponential-type distributions. *Nonlinear Analysis - Modelling and Control,* 2019, 24, 297-313. 4. R. Leipus, **J. Šiaulys,** On a closure property of convolution equivalent class of distributions. *Journal of Mathematical Analysis and Applications,* 2020, 490, 124226. 5. M. Dirma, S. Paukštys, **J. Šiaulys**, Tails of the moments for sums with dominatedly varying random summands. *Mathematics,* 2021, 9, 824.

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Board Chairman – assoc. prof. dr. Kristina Lapin