Course description

Course title	Course code
Probability theory and mathematical statistics	

Lecturer	Department where the course is delivered
Prof. Jonas Šiaulys	Department of Mathematical Analysis Faculty of Mathematics and Informatics Naugarduko St. 24, LT-03225 Vilnius, Lithuania

Cycle		Type of course
Second		Compulsory

Mode of delivery	Semester or period when the course is delivered	Language of instruction
Face-to-face	1st semester (Fall)	Lithuanian, English

Prerequisites and corequisites	
Prerequisites: Basic knowledge of mathematical analysis and probability theory.	**Corequisites (if any):**

Number of ECTS credits	Student's workload	Contact hours	Individual work hours
10	250	96	154

Course objectives: program competences to be developed		
Acquaintance with the advanced concepts, methods, and problems of probability theory and mathematical statistics usable for theory and practice.		
Learning outcomes **At the end of the course a student should:**	**Learning methods**	**Assessment methods**
- Know the main objects of probability theory under consideration; - Have advanced insight into probability concepts such as a random element, expectation, conditional expectation, characteristic function, random walk, martingale; - Have advanced insight into the main concepts of mathematical statistics such as a sample, parameter estimation, confidence intervals, linear regression; - Be able to analyze basic properties of the probability theory and mathematical statistics objects.	Problematic lecture, case analysis	Written exam
- Be able to apply the statements of probability theory and mathematical statistics; - Be able to construct the proofs of various assertions from probability theory.	Discussion lecture, concept maps, demonstration, case by case analysis.	Written exam
- Be able to prove the particular statements of the probability theory;	Debate, demonstration, preparation of readiness	Presentation

- Be able to select suitable methods to solve various problems in probability theory through checking and critiquing; - Be able to select appropriate methods for particular statistical tasks.		

Course content: breakdown of the course	**Contact hours**					**Individual work hours and assignments**	
	Lectures	Practical training	Seminars	Consultations	**Total contact hours**	**Individual work hours**	**Assignments**
Elementary probability theory	2	2			**4**	**4**	Study the first chapter of textbook [1], solve homework problems
Kolmogorov axioms, σ-algebra constructions in complex spaces.	2	2			**4**	**6**	Study §1 and §2 of the second chapter of textbook [1], solve homework problems
Introduction of probability measures on various measurable spaces.	2	2			**4**	**6**	Study §3 of the second chapter of textbook [1], solve homework problems
Random variables and random elements.	2	2			**4**	**6**	Study §4 and §5 of the second chapter of textbook [1], solve homework problems.
The expectation of a random variable and its properties	2	4			**6**	**8**	Solve the assigned exercises, repeat the expectation properties according to §6 of the second chapter of textbook [1].
Conditional expectations and their properties	2	4			**6**	**8**	Study §7 of the second chapter of textbook [1], solve homework problems
Transformations of random variables and random elements	2	2			**4**	**6**	Study §8 of the second chapter of textbook [1], solve homework problems.
Preparation and presentation of readiness			2	2	**3**	**9**	
The first midterm exam			2	1	**3**	**15**	Review the first part of the course.
Various kinds of convergence of sequences of random variables	2	2			**4**	**6**	Study §10 of the second chapter of textbook [1], solve homework problems.
Characteristic functions, the method of characteristic functions.	2	2			**4**	**6**	Study §12 of the second chapter and §3 of the third chapter of textbook [1], solve homework problems
Simple random walk	2	2			**4**	**6**	Study §9,10 of the first chapter of textbook [1],

							solve homework problems
Markov chains	2	2			**4**	**6**	Study §12 of the first chapter of textbook [1], solve homework problems
Random walk on Markov chains	2	2			**4**	**6**	Study §12 of the first chapter and §2,3 of the last chapter of textbook [1], solve homework problems
Discrete time martingales, the method of martingales.	2	4			**6**	**8**	Study §1, §3 and §4 of the 8th chapter of textbook [1], solve homework problems.
Population and sample, random sampling characteristics	2	2			**4**	**4**	Study §2.1-§2.3 of textbook [2], solve homework problems.
Order statistics of random sampling	2	2			**4**	**4**	Study §2.1 and §5.3 of textbook [2], solve homework problems.
Confidence intervals	2	2			**4**	**4**	Study §2.4 and §7.1-§7.5 of textbook [2], solve homework problems.
Testing hypotheses	2	2			**4**	**4**	Study §2.4 and §6.1-§6.4 of textbook [2], solve homework problems
Linear regression and correlation	2	2			**4**	**4**	Study §5.4 and §5.5 of textbook [2], solve homework problems.
Nonparametric tests	2	2			**4**	**4**	Study §6.5 of textbook [2], solve homework problems
Preparation and presentation of readiness			2	2	**3**	**9**	
The second midterm exam			2	1	**3**	**15**	Review the second part of the course.
Total	**38**	**44**	**8**	**6**	**96**	**154**	

Assessment strategy	**Weight**	**Time of assessment**	**Criteria**
General assessment strategy. A 10 point rating system is applied. It is possible to get 40 points on the first midterm exam. The same is possible on the second midterm exam. Additional 20 points can be collected for an individual or group self-study presentation. All collected points are added and divided by 10.			
The first midterm exam	40%	During the semester	In this exam, students are tested on the material from the first half of the semester. Typically, the exam consists of one easy theoretical question (5 points), one hard theoretical question (10 points), and a long multi-stage exercise (25 points). To answer an easy theoretical question, a student should formulate some definition, theorem, or explain some concept. The answer to this question is assessed strictly: *the student knows an appropriate definition or concept (5 points); the student does not know the appropriate definition or concept (0 points).* A hard theoretical question is the proof of some assertion known from the syllabus. Given proof is assessed in a standard way: *the student has not started proving the statement (0 points); the statement remains unproven, but the student made a few required correct steps of the proof (1-4 points); the assertion has been proved with large defects (5-6 points); the proof of the statement was presented with minor deficiencies (7-8 points); the proof of the statement was presented without any defects, all important places of the proof are fully*

			explained (9-10 points). A long multi-stage exercise usually consists of five parts. In each of these parts, a student needs to find some characteristic of the same discrete time risk model. Each part of the exercise is assessed in points from 0 to 5 in a standard way: *the student has not tried to find the desired model characteristic (0 points); the student in search of the required characteristic has made several essential errors (1-2 points); while finding the desired characteristic, the student made a few minor, e.g., arithmetic, errors (3-4 points); the student found correctly the desired characteristic of the model, all calculations and derivations are correct and accurate (5 points).*
The second midterm exam	40%	At the end of the semester	In this exam, students are tested on the material from the second half of the semester. The second midterm exam's composition and assessment are similar to the composition and the assessment of the first midterm exam.
Presentation	20%	During the semester	At the beginning of the semester, all students individually receive a task for readiness. The task consists of a theoretical problem, a complicated exercise, or of a practical problem. Topics are coordinated with students. Most of the topics require reading supplementary material. When the agreed time comes, each student presents a task done in electronic form. Successfully completed tasks are presented during the seminars. One presentation takes approximately 15 minutes.

Author	**Publication year**	**Title**	**Volume and/or publication number**	**Publication place and publisher**
Required reading				
A.N Shiryaev	1996	Probability		Springer
Shao Jun	2003	Mathematical Statistics		Springer
Recommended reading				
V.V.Petrov	1995	Limit Theorems of Probability Theory		Clarendon Press, Oxford
G. Grimmett, D. Stirzaker	2001	One Thousand Exercises in Probability		Oxford University Press