



COURSE UNIT (MODULE) DESCRIPTION

Course unit (module) title	Code
Modelling of stochastic processes	

Academic staff	Core academic unit(s)
Coordinator: Assoc. Prof. Dr. Jevgenij Chmeliiov Next(s): None	Faculty of Physics, Institute of Chemical Physics

Study cycle	Type of the course unit
Second	Optional

Mode of delivery	Semester or period	Mode of delivery
Contact (auditorium)	Autumn semester	Lithuanian, English

Requisites	
Prerequisites: Background in higher mathematics, statistical physics and programming	Related requirements (if any):

Number of ECTS credits allocated	Student's workload (total)	Contact hours	Individual work
5	140	48	92

Purpose of the course unit
To introduce the peculiarities of modelling of various stochastic processes relevant in physical sciences, to learn how to describe these physical phenomena using mathematical equations and how to solve them using analytical and/or numerical methods.

Learning outcomes of the course unit	Teaching and learning methods	Assessment methods
The student: - will understand the essence of modeling - will demonstrate knowledge of various aspects of stochastic processes: the similarity and difference between stochastic and deterministic descriptions, simple population models and their modifications to describe various physical phenomena - will be able to write down the formulated model in mathematical form, relate model variables to the physical observables, determine the parameters of the model - will be ready to independently analyze various models of physical phenomena, seek knowledge in the relevant field of science	Problem teaching, demonstration (derivations of equations, algorithm schemes) active teaching methods (group discussion), research methods (systematization of information and preparation of a report)	Presentation during the seminar
- will be able to solve the equations of some models analytically - will be able to convert mathematical equations into computer software code, choose the right method to numerically solve a specific task, evaluate the required accuracy of the results and the speed of execution of the solution algorithm	Homework (problem solving, writing computer programs, presenting the results), active teaching methods (discussing the results in a group discussion)	Solutions to homework tasks and self-developed computer programs
- will be able to independently formulate a model for the description of the observed physical phenomenon - will be able to apply the reverse task regularization procedure and determine the parameters of the model by combining the dynamic variables of the model with the measurable physical quantities - will be able to critically assess the applicability of different models	Independent research, presentation and discussion of research results	Reports

Content	Contact hours							Individual work: time and assignments	
	Lectures	Tutorials	Seminars	Workshops	Laboratory work	Internship	Contact hours, total	Individual work	Tasks for individual work
1. Introduction. The interface between theory and experiment. Physical models and numerical experiment. Structure and need for modeling. Classification of models.	2	0	0	0	0	0	2	0	
2. Basic concepts of modeling of stochastic processes: Probability, conditional probability, weighted probability of the stochastic variable, probability density function, n -th order moments. The random variable and its various distributions. The generating function, its application to calculate weighted probabilities, expectation values, and variances of various distributions. Monte-Carlo modeling algorithm.	2	0	1	0	0	0	3	6	Literature analysis, practical tasks
3. Discrete-time Markov chains: Markov process, transition matrix, Chapman–Kolmogorov equation. Stationary weighted probabilities. The mean-time of the first return, the average first-passage time between two nodes. One-dimensional random walk, probability of absorption at the left and right edges, average duration of random walk. Examples of the application of Markov chains: population growth/extinction models, epidemic models.	6	0	3	0	0	0	9	16	Literature analysis, practical tasks
4. Branching processes: Galton–Watson process, its generating function, probability of population extinction, mean size of the n -th generation population	2	0	1	0	0	0	3	4	Literature analysis, practical tasks
5. Continuous-time Markov Chains: Poisson's Process. Kolmogorov's equations, the Master equation. Stationary solution and mean first-return time. Generating function, probability of survival, mean lifetime. Continuous-time population models. Deterministic and stochastic dynamics. Nonlinear population models	10	0	6	0	0	0	16	24	Literature analysis, practical tasks
6. Diffusion processes. One-dimensional diffusion in infinite medium. Diffusion in the finite interval between the two absorbing sides, Kolmogorov's equation, Green's function, probability of survival, mean lifetime	4	0	2	0	0	0	6	10	Literature analysis, practical tasks
7. Anomalous diffusion: continuous-time random walk, subdiffusion and superdiffusion processes, their peculiarities, examples of application. Riemann-Liouville fractional differential operator. Diffusion in disordered systems.	4	0	2	0	0	0	6	4	Literature analysis, practical tasks

8. Partial-derivative models: structured-age population model, propagating wavefronts	2	0	0	0	0	0	2	2	Literature analysis, practical tasks
9. Project task: to propose a model that could describe the data provided. Write down all the necessary mathematical equations, choose the right algorithm for solving them. When performing numerical modeling, justify the suitability of this model and determine the parameters of the proposed model. Discuss the physical meaning of the obtained parameters, prepare a report.	0	0	1	0	0	0	1	26	Literature analysis, practical tasks
Total	32	0	16	0	0	0	48	92	

Assessment strategy	Weight %	Deadline	Assessment criteria
Homework	30	Throughout the semester	Solutions to homework tasks and independently prepared computer programs.
Presentation at the seminar	10	Throughout the semester	Criteria for evaluating the reports prepared by the students: 1. Comprehensiveness of familiarity with the presented topic 2. Ability to answer questions related to the topic 3. Ability to participate in a discussion about issues related to the topic.
Project modeling task	30	In the second part of the semester	The submitted project task report will be evaluated according to the following criteria: 1. Validity of the proposed model 2. Ability to formulate the proposed model in mathematical language, choose appropriate solution algorithms, perform numerical modeling 3. Ability to describe the obtained results, discuss the meaning and reliability of the obtained parameters of the model, and indicate the limits of validity of the model
Exam	30	Session	The exam consists of theoretical questions and practical tasks.

Author (-s)	Publishing year	Title	Issue of a periodical or volume of a publication	Publishing house or web link
Compulsory Literature				
L. J. S. Allen	2015	An Introduction to Stochastic Processes with Applications to Biology, 2nd ed.		CRC Press, Taylor & Francis Group, Boca Raton
N. G. van Kampen	2004	Stochastic Processes in Physics and Chemistry		North-Holland, Amsterdam
Further Reading				
G. de Vries, T. Hilgen, M. Lewis, J. Müller, B. Schönfish	2006	A Course in Mathematical Biology. Quantitative Modelling with Mathematical and Computational methods		SIAM Society for Industrial and Applied Mathematics, Philadelphia
C. W. Gardiner	2004	Handbook of Stochastic methods. For Physics, Chemistry and the Natural Sciences		Springer, Berlin
D. W. Heerman	1990	Computer simulations methods in theoretical physics		Springer-Verlag
J. W. Haus, K. W. Kehr	1987	Diffusion in Regular and Disordered Lattices	263-408, vol. 150	Physics Reports, Elsevier Science Publishers