DOCTORAL (PHD) STUDIES
COURSE DESCRIPTION

Course title	Field of science	Faculty	Institute
Mathematical Analysis	Mathematics (N 001)	Faculty of Mathematics and Informatics	Institute of Applied Mathematics
Study method	Number of credits	Study method	Number of credits
Lectures	2	Consultations	1
Individual work	7	Seminars	0

Course summary

The course consists of three modules. To pass the course one needs to pass each module.

Annotation of module 1. (Contact hours: 20 academic hours).

1. The history of mathematical analysis and elements of the set theory.
2. The axioms of a set of natural numbers.
3. Construction of a set of real numbers.
4. Completeness of a set of real numbers and its equivalent forms.
5. Classes of functions. From differentiable functions to rough functions.
6. Variation of a function and its generalizations.
7. Integration: Cauchy, Riemann, Lebesgue, Henstock-Kurzweil and other integrals.

Literature:

1. H.N. Jahnke (Ed.). A History of Analysis. London Mathematical Society, 2003.
2. E.D. Bloch. The Real Numbers and Real Analysis. Springer, 2011.
3. J. Appell, J. Banas, N. Merentes. Bounded Variation and Around. De Gruyter, 2014.
4. F.E. Burk. A Garden of Integrals. The Mathematical Association of America, 2007.

Annotation of module 2. (Contact hours: 24 academic hours).

1. Middle value Theorems and applications.
2. Inequalities. Jensen‘s inequality and applications.
3. Stolz Cesaro theorem and corollaries.
4. Sequences, series and infinite multipications. Raabe‘s test.
5. Functional equations.
6. Integrals, Helder‘s inequality.
7. Some problems of optimization in analysis.

Literature:

1. K.Kedlaya, Putnam mathematical competition, https://kskedlaya.org/putnam-archive/
2. Vojtech Jarnik mathematical competition, https://vjimc.osu.cz/problems
3. International math competitions for university students, https://www.imc-math.org.uk/
4. Demidovich, Problems of analysis, Moscow, 2002 (in russian).

Annotation of module 3. (Contact hours: 20 academic hours).

1. Analysis on subsets of an *m*-dimensional space (reminding): interior, closure and boundary of a set, open and closed sets, continuous functions, compact sets, connected sets.
2. Manifolds: smooth vector functions and their differentiation, parameterizations, elementary manifolds and manifolds in an *m*-dimensional space, tangent and normal vectors of a manifold, regular part of the boundary of a domain.
3. Multivectors: exterior product of vectors, multivectors, orientation of a manifols at a point, orientable manifolds, oriented manifolds, differential forms, differential of a function, exterior derivative of a form.
4. Integration of forms: Hausdorff measures, integration with respect to Hausdorff measures, domains with almost regular boundary, integration of forms, outward orientation of the boundary of a domain, the Stokes theorem.

Literature:

1. V. Kazakevičius. Lecture notes on the Stokes theorem, 2021. stokes.pdf (vu.lt)
2. L. Schwartz. Cours d'analyse 2, 1981.

Total number of contact hours: 64 academic hours.

Consulting teacher	Scientific degree	Pedagogical name	Main publications in the field of science of the last 5 year period
Vytautas Kazakevičius	Dr.	Assoc. Prof.	1. Kazakevičius, Vytautas. Subadditive ergodic theorem for double sequences // Journal of theoretical probability, 2021, vol. 34, p. 307-330. 2. Kazakevičiūtė, Agnė; Kazakevičius, Vytautas; Olivo, Malini. Conditions for existence of uniformly consistent classifiers // IEEE Transactions on information theory, 2017, vol. 63, issue 6, p. 3425-3432.
Rimas Norvaiša	Habil dr.	Prof.	1. R. Norvaiša, A. Račkauskas. Uniform asymptotic normality of weighted sums of short-memory linear processes. Journal of Applied Probability, vol. 57, Issue 1, 2020, pp. 174-195. 2. R. Norvaiša, A. Račkauskas. Uniform asymptotic normality of self-normalized weighted sums of random variables. Lithuanian Mathematical Journal, vol. 59, No. 4, 2019, pp. 575-594. 3. V. Butkus, R. Norvaiša. Computation of p-variation. Lithuanian Mathematical Journal, Vol. 58, No. 4, 2018, 360–378. DOI 10.1007/s10986-018-9414-3.
Gintaras Puriuškis	Dr.	Assoc. Prof.	1. Dubickas, Artūras; Puriuškis, Gintaras. On the minimum of certain functional related to the Schrödinger equation. Electronic journal of qualitative theory of differential equations. Szeged: Szegedi Tudomanyegyetem, 2013, no 8. p. [1-21]. [Peer-reviewed journal] ISSN: 1417-3875. 2. Ivanauskas, Feliksas; Puriuškis, Gintaras. Blow-up of the solution of a nonlinear Schrödinger equation system with periodic boundary conditions. Nonlinear analysis: modelling and control. Vilnius: Institute of Mathematics and Informatics, 2013, vol. 18, no. 1. p. 53-65. [Peer-reviewed journal] ISSN: 1392-5113

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Board Chairman – assoc. prof. dr. Kristina Lapin