## DOCTORAL (PHD) STUDIES COURSE DESCRIPTION

Course title	Field of science	Faculty	Institute
Stochastic Differential	Mathematics	Faculty of	Institute of Data
Equations	(N 001)	Mathematics and Informatics	Science and Digital Technologies
Study method	Number of credits	Study method	Number of credits
Lectures	0	Consultations	1
Individual work	4	Seminars	0

## Course summary

- 1. *Continuous local martingales and stochastic integrals*. Properties of quadratic variation, stochastic integrals and Ito formula for continuous semimartingales.
- 2. *Continuous martingales and Brownian motion*. Martingale characterization of Brownian motion, random timechange, Cameron–Martin and Girsanov's theorems.
- 3. *Stochastic differential equations*. Linear stochastic differential equations, Ornstein–Uhlenbeck processes, existence and uniqueness of strong solution.
- 4. Markovian property of the solution of stochastic differential equation and infinitesimal generator.
- 5. *Weak solutions of stochastic differential equation.* Weak solutions and martingale problem, Yamada-Watanabe theorem.
- 6. *Kalman-Bucy filter*.
- 7. *Parabolic equations*. Heat equation. Feynman–Kac formula, the stochastic solution to a Cauchy problem for a parabolic equation.
- 8. Elliptic equations. Stochastic solutions of the Dirichlet problem for elliptic equations.

## Main literature

1.V. Mackevičius, Introduction to stochastic analysis: integrals and differential equations, Wiley-ISTE, 2011.

- 2. J.-F. Le Gall, Brownian motion, martingales, and stochastic calculus, Springer, 2016.
- 3. Kallenberg, Foundations of Modern Probability, Springer, 1997.

4. B.Oksendal. Stochastic Differential Equations, An Introduction with Applications, Springer, Berlin et all., 5 ed., 1998.

5. Karatzas, S.E. Shreve, Brownian Motion and Stochastic Calculus, Springer-Verlag, 1991

6. A. Friedman. Stochastic Differential Equations and Applications. I, II, Academic Press, 1975.

Consulting teacher	Scientific	Pedagogical	Main publications in the field of science of the last 5
	degree	name	year period
Kęstutis Kubilius	Habil. dr.	Prof.	<ol> <li>K. Kubilius, Yu. Mishura, K. Ralchenko. Parameter estimation in fractional diffusion models. Cham: Springer International Publishing, 2017. XIX, 390 p. (Bocconi &amp; Springer Series).</li> <li>K. Kubilius, A. Medžiūnas, Positive solutions of the fractional SDEs with non-Lipschitz diffusion coefficient, Mathematics, 2021, 9(1), 1–14.</li> <li>K. Kubilius, Estimation of the Hurst index of the solutions of fractional SDE with locally Lipschitz drift. Nonlinear Anal. Model. Control 2020, 25, 1059–1078</li> <li>K. Kubilius, CLT for quadratic variation of Gaussian processes and its application to the estimation of the Orey index, Statistics &amp; Probability Letters, 165:108845, 2020.</li> <li>K. Kubilius, V. Skorniakov. A short note on a class of statistics for estimation of the Hurst index of fractional Brownian motion. Statistics &amp; probability letters. 2017, Vol. 121, p. 78-82.</li> </ol>

6. K. Kubilius, V. Skorniakov, K. Ralchenko, <i>The rate</i>
of convergence of the Hurst index estimate for a stochastic differential equation. Nonlinear analysis:
modelling and control. 2017, Vol. 22, No. 2, p. 273-
284

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Board Chairman – assoc. prof. dr. Kristina Lapin